Low Energy QCD
and ChPT Tests
in the NA48/2 Experiment

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New Trends In High-Energy Physics
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Outline

› NA48/2 Experimental Setup

› Low Energy QCD:
  › $K_{e4}$: Form Factors and $\pi\pi$ Scattering Length
  › $K^\pm \to \pi^\pm\pi^0\pi^0$: “Cusp” Effect and $\pi\pi$ Scattering Length

› ChPT Tests:
  › $K^\pm \to \pi^\pm\pi^0\gamma$: First Evidence of Interference Term
  › $K^\pm \to \pi^\pm e^+e^-\gamma$: First Observation
NA48/2
Experimental Setup
**History**

**NA48 (1997–2000):** Direct CP-Violation in neutral $K$

$\text{Re}(\varepsilon'/\varepsilon) = (14.7 \pm 2.2) \cdot 10^{-4}$

**NA48/1 (2002):** Rare $K_S$ decays

$\text{BR}(K_S \rightarrow \pi^0 e^+ e^-) = (5.8^{+2.8}_{-2.3} \pm 0.8) \cdot 10^{-9}$

$\text{BR}(K_S \rightarrow \pi^0 \mu^+ \mu^-) = (2.8^{+1.5}_{-1.2} \pm 0.2) \cdot 10^{-9}$

**NA48/2 (2003–2004):** Direct CP-Violation in charged $K$

$A_g(K^\pm \rightarrow \pi^\pm \pi^+ \pi^-) = (-1.5 \pm 2.1) \cdot 10^{-4}$

$A_g(K^\pm \rightarrow \pi^\mp \pi^0 \pi^0) = (1.8 \pm 1.8) \cdot 10^{-4}$

…and many other results on kaon and hyperon decays
Simultaneous Beam

2-3M K/spill (π/K ~ 10) π decay products stay in pipe
Flux ratio: K+/K- ~ 1.8

Simultaneous K+ and K- beams:
large charge symmetrization
of experimental conditions

Beams coincide within ~1mm
all along 114m decay volume

~7·10^{11} p/spill
400 GeV/c
Front-end achromat:
Momentum selection

\[ P_K = (60 \pm 3) \text{ GeV/c} \]

Quadrupole, Quadruplet:
Focusing μ sweeping

Second achromat:
Cleaning Beam spectrometer

\[ \delta P_K / P_K = 0.7\% \]
\[ \delta x, y \sim 100 \mu m \]

Width ~ 5 mm
K+/K- ~ 1 mm
**Detector**

**Magnetic spectrometer (4 DCHs):**
- 4 view / DCH -> high efficiency
- $\sigma_p/P = 1.0\% + 0.044\%\cdot P \ [\text{GeV/c}]$

**Hodoscope:**
- Fast trigger
- $\sigma_+ = 150\text{ps}$

**Electromagnetic calorimeter (LKr):**
- High granularity, quasi-homogeneous
- $\sigma_E/E = 3.2\%/\sqrt{E} + 9\%/E + 0.42\% \ [\text{GeV}]$

**Hadron calorimeter, muon veto and photon vetoes**

**Trigger:**
- Fast hardware trigger (L1): hodoscope & DCHs multiplicity
- Level 2 trigger (L2): on-line processing of DCHs & LKr information
Data Taking

Run periods:
> 2003: ~ 50 days
> 2004: ~ 60 days

Total statistics in 2 years:
> $K^\pm \rightarrow \pi^\pm \pi^+\pi^-$: ~ $4 \cdot 10^9$
> $K^\pm \rightarrow \pi^\pm \pi^0\pi^0$: ~ $1 \cdot 10^8$

-> >200 TB of data recorded

A view of the NA48/2 beam line

Rare $K^\pm$ decays can be measured down to $\text{BR} \sim 10^{-9}$
Low Energy QCD: $K_{e4}$ Decay
The $K^\pm \to \pi^+\pi^-e^\pm\nu$ ($K_{e4}$) dynamic is fully described by 5 variables (Cabibbo-Maksymovicz):

$$M_{\pi\pi}^2, M_{e\nu}^2, \cos\theta_{\pi}, \cos\theta_{e} \text{ and } \phi$$

The transition amplitude can be written using 2 axial and 1 vector Form Factors that can be developed in a partial wave expansion:

F = $F_s e^{i\delta_s} + F_p e^{i\delta_p} \cos\theta_{\pi} + \text{terms}_{d\text{-wave}}$

G = $G_p e^{i\delta_p} + \text{terms}_{d\text{-wave}}$

H = $H_p e^{i\delta_p} + \text{terms}_{d\text{-wave}}$

The Form Factors can be expanded as a function of $q^2=(M_{\pi\pi}^2/4m_{\pi}^2-1)$ and $M_{e\nu}^2$:

$$F_s = f_s + f_s' q^2 + f_s'' q^4 + f_e' (M_{e\nu}^2/4m_{\pi}^2) + ...$$

$$F_p = f_p + f_p' q^2 + ...$$

$$G_p = g_p + g_p' q^2 + ...$$

$$H_p = h_p + h_p' q^2 + ...$$

$F_s, F_p, G_p, H_p$ and $\delta=\delta_s - \delta_p$ used as fit parameters
Selection and Background

**Signal selection:**
- 3 charged tracks
- 2 opposite sign $\pi$s
- 1 e: LKr & DCH info E/P
- 1 $\nu$: some missing energy & $P_\perp$

**Main background sources:**
- $K^\pm \rightarrow \pi^\pm \pi^+ \pi^-$ decay with $\pi \rightarrow e\nu$ (dominant) or $\pi$ mis-ID as e
- $K^\pm \rightarrow \pi^\pm \pi^0(\pi^0)$ decay with $\pi^0 \rightarrow e^+e^-\gamma$ and e mis-ID as $\pi + \gamma$s undetected
- Background is studied using the electron “wrong sign” events (assuming $\Delta Q=\Delta S$ and total charge $\pm 1$) and cross checked with MC

2003: 677500 events
Total background can be kept @ ~0.5% level
Using iso-populated bins in the 5-D space of the C.M. variables one defines a grid of:

\[10(M_{\pi\pi}) \times 5(M_{e\nu}) \times 5(\cos\theta_e) \times 5(\cos\theta_\pi) \times 12(\phi) = 15000 \text{ boxes}\]

The set of Form Factor values is used to minimize a log-likelihood estimator well suited for small numbers of data event/bin and taking into account the statistics of the simulation (simulated and expected events/bin)

Assuming constant Form Factors over single boxes, ten independent fits in \(M_{\pi\pi}\) bins have been performed to get model independent results
Results (I)

\[ F_\pi = 1 + f_0^{'} / f_\pi q^2 + f_1^{'} / f_\pi q^4 \]

- Quadratic in \( q^2 \)

\[ g_\rho / f_\pi + g_\rho^{'} / f_\pi q^2 \]

- Linear in \( q^2 \)

\[ F_p(q^2) \]

- First measurement of \( F_p \neq 0 \)

\[ H_p(q^2) \]

- No linear term \( (h_p^{'}) \)

Preliminary
Results (II)

| $f_s'/f_s$ | 0.165 ± 0.011 ± 0.006 |
| $f_s''/f_s$ | -0.092 ± 0.011 ± 0.007 |
| $f_e'/f_s$  | 0.081 ± 0.011 ± 0.008 |
| $f_p/f_s$   | -0.048 ± 0.004 ± 0.004 |
| $g_p/f_s$   | 0.873 ± 0.013 ± 0.012 |
| $g_p'/f_s$  | 0.081 ± 0.022 ± 0.014 |
| $h_p/f_s$   | -0.411 ± 0.019 ± 0.007 |

Systematics checks:

- Acceptance
- Background
- Particles ID
- Radiative corrections

- All the Form Factors are measured relatively to $f_s$ (no BR measurement)
- Because of different beam geometries for $K^+$ and $K^-$, the event samples are fitted separately and the results combined according to their statistical precision ($K^+/K^- \sim 1.8$)
- First evidence of $f_p \neq 0$ and $f_e' \neq 0$
- Neglected $M_{ev}$ dependence of the normalization
- The Form Factors are measured at level of <5% of relative precision while the slopes at ~15% relative precision (factor 2÷3 improvement wrt. previous measurements)
The extraction of the $\pi\pi$ scattering lengths from the $\delta = \delta_s - \delta_p$ phase shift needs external theoretical and experimental data inputs.

The Roy equations provide this relation between $\delta$ and $(a_0, a_2)$ near threshold, extrapolating from the $M_{\pi\pi} > 0.8$ GeV/$c^2$ region. The precision of these data defines the width of the Universal Band in the $(a_0, a_2)$ plane.

The fit of the experimental points using the Roy equations in the Universal Band allows to extract the $a_0$ and $a_2$ values.
Minimizing the $\chi^2$ in the 2-D fit it's possible to identify the favoured solution (and the corresponding ellipse)

Single parameter fit ($a_2$ constrained to the center line of the UB):

- $a_0 \cdot m_{\pi^+} = 0.256 \pm 0.006_{\text{stat}} \pm 0.005_{\text{syst}}$
- $(a_2 \cdot m_{\pi^+} = -0.031 \pm 0.001_{\text{stat}} \pm 0.001_{\text{syst}})$

Two parameters fit:

- $a_0 \cdot m_{\pi^+} = 0.233 \pm 0.016_{\text{stat}} \pm 0.012_{\text{syst}}$
- $a_2 \cdot m_{\pi^+} = -0.047 \pm 0.011_{\text{stat}} \pm 0.008_{\text{syst}}$

- The E865 and NA48/2 results agreement is marginal (mainly due to the last $\delta$ point in E865)
- The correlation between $a_0$ and $a_2$ is $\sim 96\%$ (similar for both experiment)
(a₀, a₂) Plane (II)

Following recent developments one can correct the measured $K_{e4}$ phases for isospin symmetry breaking effect before extracting $a₀$ (the correction is $\sim 10\div 12$ mrad (negative))

- Using preliminary isospin corrections, both $a₀$ and $a₂$ values decrease (by $\sim 0.02$ and $\sim 0.004$ respectively) with statistical and systematic errors unchanged
- The new values would then be in very good agreement with the preferred ChPT prediction ($a₀ = 0.220$, $a₂ = -0.0444$) and the most recent lattice calculations ($a₂ = -0.04330 \pm 0.00042$)

Both bands shift left and down in the (a₀, a₂) plane
**Signal selection:**

- 1 e in the DCHs
- 4 γs in the LKr
- π⁰ mass constraints
- 1 ν: missing P⁺

**Main background sources:**

- \( K^± \rightarrow π^±π^0π^0 \) with a mis-ID π±
- \( K^± \rightarrow π^0e^±γ \) with 1 accidental γ and the 2 γs faking a π⁰

\[ \text{BR} = (2.587 \pm 0.026_{\text{stat}} \pm 0.019_{\text{syst}} \pm 0.029_{\text{norm}}) \cdot 10^{-5} \]

\[ f_s'/f_s = 0.129 \pm 0.036_{\text{stat}} \pm 0.020_{\text{syst}} \]

\[ f_s''/f_s = -0.040 \pm 0.034_{\text{stat}} \pm 0.020_{\text{syst}} \]

- \( f_e' \) consistent with 0 within the present statistics

*2003+2004: \( \sim 38000 \) events with \( \sim 2\%\) of residual background*
Low Energy QCD:

$K^\pm \rightarrow \pi^\pm \pi^0\pi^0$ Decay
> From $K^\pm \to \pi^+\pi^-\pi^0$ decay (partial sample of 2003 data) we observed an anomaly in the $M_{00}^2$ invariant mass distribution in the region around $M_{00}^2 = (2m_{\pi^+})^2 = 0.07792 \text{ (GeV}^2/\text{c}^4) \text{.}$

> This anomaly has been interpreted as a final state charge exchange scattering process of $K^\pm \to \pi^+\pi^-\pi^0$ ($\pi^+\pi^- \to \pi^0\pi^0$).

> The parameter $a_0-a_2$ (difference between the S-wave $\pi\pi$ scattering lengths in the isospin I=0 and I=2 states) can be precisely measured using this sudden anomaly (“cusp”).
Interpretation (I)

Re-scattering model: two amplitudes contribute to $K^+ \rightarrow \pi^\pm \pi^0 \pi^0$

$$\mathcal{M}(K^+ \rightarrow \pi^\pm \pi^0 \pi^0) = M_0 + M_1$$

- $M_0$: Direct emission
- $M_1$: Charge exchange in final state of $K^+ \rightarrow \pi^\pm \pi^\pm \pi^- (\pi^+ \pi^- \rightarrow \pi^0 \pi^0)$

The singularity in the invariant mass spectrum at $\pi^+ \pi^-$ threshold is mainly caused by the destructive interference of $M_0$ and $M_1$

The effect is present below the threshold and not above it (re-scattering model at one-loop (N. Cabibbo: PRL 93 (2004) 121801))
More complete formulation of the model including all re-scattering processes at one-loop and two-loop level (N. Cabibbo and G. Isidori: JHEP 0503 (2005) 21) has been used to extract NA48/2 results.
Results (I)

**Systematic checks:** acceptance determination, trigger efficiency and fitting interval

\[ g_0 = 0.645 \pm 0.004_{\text{stat}} \pm 0.009_{\text{syst}} \]
\[ h' = -0.047 \pm 0.012_{\text{stat}} \pm 0.011_{\text{syst}} \]
\[ (a_0 - a_2) \cdot m_{\pi^+} = 0.268 \pm 0.010_{\text{stat}} \pm 0.004_{\text{syst}} \pm 0.013_{\text{ext}} \]
\[ a_2 \cdot m_{\pi^+} = -0.041 \pm 0.022_{\text{stat}} \pm 0.014_{\text{syst}} \]

Predictions in ChPT (PLB 488 (2000) 261):
- \( (a_0 - a_2) \cdot m_{\pi^+} = 0.265 \pm 0.004 \)
- \( a_2 \cdot m_{\pi^+} = -0.0444 \pm 0.0010 \)

Fit imposing ChPT constraint between \( a_0 \) and \( a_2 \) (PRL 86 (2001) 5008)

\[ a_0 \cdot m_{\pi^+} = 0.220 \pm 0.006_{\text{stat}} \pm 0.004_{\text{syst}} \pm 0.011_{\text{ext}} \]
\[ (a_0 - a_2) \cdot m_{\pi^+} = 0.264 \pm 0.006_{\text{stat}} \pm 0.004_{\text{syst}} \pm 0.013_{\text{ext}} \]

**Results (II)**

Fitting Dalitz plot above cusp finds evidence for $k' > 0$ term

$$M^2_{++00} = 1 + \frac{1}{2} g_0 u + \frac{1}{2} h' u^2 + \frac{1}{2} k' v^2 + ...$$

Change of Dalitz variables, from $(s_3, s_2-s_1)$ to $(s_3, \cos \theta)$. Define $\theta$ as angle between $\pi^\pm$ and $\pi^0$ in $\pi^0\pi^0$ rest frame:

$g_0$ and $h'$ change (2% and 25% respectively) but no change in $(a_0-a_2) \cdot m_{\pi^+}$ and $a_2 \cdot m_{\pi^+}$

$$k' = 0.0097 \pm 0.0003_{\text{stat}} \pm 0.0008_{\text{syst}}$$
ChPT Tests:

\[ K^\pm \rightarrow \pi^\pm \pi^0 \gamma \text{ Decay} \]
\[
\frac{d\Gamma^\pm}{dW} \approx \left( \frac{d\Gamma^\pm}{dW} \right)_{IB} \left[ 1 + 2 \left( \frac{m_\pi}{m_K} \right)^2 \cdot W^2 \cdot |E| \cos((\delta_1 - \delta_0) \pm \phi) + \left( \frac{m_\pi}{m_K} \right)^4 \cdot W^4 \cdot (|E|^2 + |M|^2) \right]
\]

\[W^2 = \frac{\left( P_K^* \cdot P_\gamma^* \right) \left( P_\pi^* \cdot P_\gamma^* \right)}{\left( m_K m_\pi \right)^2}\]

**PDG (55 \text{ MeV} < T^*_\pi < 90 \text{ MeV})**

- **IB**: $(2.75 \pm 0.15) \cdot 10^{-4}$
- **DE**: $(4.4 \pm 0.8) \cdot 10^{-6}$
- **INT**: not yet measured

**All previous measurements have been performed setting INT to zero**
**Data/MC Comparison**

In the 2003 data sample (~30% of the whole statistics) \( \sim 220 \cdot 10^3 \) 

\[ K^\pm \rightarrow \pi^\pm \pi^0 \gamma \] 

have been selected:

- After trigger efficiency correction good agreement between Data and MC for \( E_\gamma \), in particular for \( E_\gamma > 5 \) GeV (used for final result)
- The ratio \( W(\text{Data})/W(\text{MC}_{IB}) \) is in good agreement for IB dominated region and clearly shows DE

![Graph showing data and fit results](image)
Use extended Maximum Likelihood for \(0.2 < W < 0.9\) to fit in the region \(0 \text{ MeV} < T^*_\pi < 80 \text{ MeV}\) (based on \(124 \cdot 10^3\) events)

\[\text{Frac}(\text{INT}) = (-2.67 \pm 0.81_{\text{stat}} \pm 0.73_{\text{syst}}\text{ syst})\%\]

\[\text{Frac}(\text{DE}) = (3.35 \pm 0.35_{\text{stat}} \pm 0.25_{\text{syst}}\text{ syst})\%\]

\(\rightarrow\) First evidence of Interference between Inner Bremsstrahlung and Direct Emission amplitudes
ChPT Tests:

$K^\pm \rightarrow \pi^\pm e^+ e^- \gamma$ Decay
**BR Measurement**

Never observed before. Naïve estimation of the BR:

\[ \text{BR}(\pi^\pm e^+e^-\gamma) = \text{BR}(\pi^\pm\gamma\gamma) \cdot 2\alpha \sim 1.6 \times 10^{-8} \]

Theoretical expectation (ChPT, Gabbiani99):

\[ \text{BR}(\pi^\pm e^+e^-\gamma) = (0.9 \pm 1.6) \times 10^{-8} \]

depending on $\hat{c}$

**Event sample (2003+2004):**
- 92 candidates events
- $1 \pm 1$ accidental background
- $5.1 \pm 1.7$ physical background

**Normalization channel:**

$K^\pm \to \pi^\pm\pi^0_D$: $14 \times 10^{-6}$ events

\[ \text{BR}(K^\pm \to \pi^\pm e^+e^-\gamma) = (1.27 \pm 0.14_{\text{stat}} \pm 0.05_{\text{syst}}) \times 10^{-8} \]
Summary (I)

> NA48/2 has improved measurements of the $K_{e4}$ Form Factors in the “charged” and “neutral” modes (5±30% relative statistical precision)

> Using a conservative theoretical approach, preliminary values of $a_0$ and $a_2$ are obtained (2-D fit):

\[
a_0 \cdot m_{\pi^+} = 0.233 \pm 0.016_{\text{stat}} \pm 0.012_{\text{syst}}
\]

\[
a_2 \cdot m_{\pi^+} = -0.047 \pm 0.011_{\text{stat}} \pm 0.008_{\text{syst}}
\]

> More elaborated theoretical corrections would bring down those values in close agreement with ChPT predictions

> A new “cusp” structure in $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$ was observed ($\pi\pi$ final state charge exchange process of $K^\pm \rightarrow \pi^\pm \pi^+ \pi^-$) which provides a new method for the extraction of the $\pi\pi$ scattering lengths:

\[
(a_0-a_2) \cdot m_{\pi^+} = 0.268 \pm 0.010_{\text{stat}} \pm 0.004_{\text{syst}} \pm 0.013_{\text{theor}}
\]

> Parameter $a_2$ directly measured for the first time even though with low accuracy:

\[
a_2 \cdot m_{\pi^+} = -0.041 \pm 0.022_{\text{stat}} \pm 0.014_{\text{syst}}
\]
The first measurement of Direct Emission and Interference terms in $K^\pm \rightarrow \pi^\pm \pi^0 \gamma$ has been performed in the region $0 \text{ MeV} < T^*_\pi < 80 \text{ MeV}$:

\[
\text{Frac}(\text{DE}) = (3.35 \pm 0.35_{\text{stat}} \pm 0.25_{\text{syst}} \%) \\
\text{Frac}(\text{INT}) = (-2.67 \pm 0.81_{\text{stat}} \pm 0.73_{\text{syst}} \%)
\]

A first evidence of a negative Interference has been found and therefore a non negligible contribution of electric term to Direct Emission amplitude.

The $K^\pm \rightarrow \pi^\pm e^+e^-\gamma$ decay has been observed for the first time:

\[
\text{BR}(K^\pm \rightarrow \pi^\pm e^+e^-\gamma) = (1.27 \pm 0.14_{\text{stat}} \pm 0.05_{\text{syst}}) \cdot 10^{-8}
\]